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NONSTABLE K-THEORY RESULTS FOR SOME AH-ALGEBRAS

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In this note we prove that "many" unital AH-algebras (which may not be of real rank zero) have cancellation, (SC) and that their K_0-group is weakly unperforated in the sense of G. A. Elliott. A new characterization of the AH-Algebras of real rank zero and with the dimensions of their local spectra uniformly bounded ($\leqq 3$) is given.

1. Introduction.

In this note we consider C^*-algebras that can be represented as inductive limits $A = \varinjlim(A_n, \Phi_{n,m})$ of what we shall call trivial locally homogeneous C^*-algebras $A_n = \bigoplus_{i=1}^{s_n} P_{n,i}M_{[n,i]}(C(X_{n,i}))P_{n,i}$, where $X_{n,i}$ are finite connected CW complexes and $P_{n,i}$ are projections. Following a terminology introduced in [4] we shall call these C^*-algebras AH algebras (approximately homogeneous). E. G. Effros raised in [8] the problem of finding suitable invariants for AH algebras. We prove that "many" unital AH algebras A have cancellation and also have (SC) (strict comparability), i.e. for any two projections p and q in A such that $\tau(p) < \tau(q)$ for any tracial state τ of A it follows that p is (Murray-von Neumann) equivalent to a proper subprojection of q (see Theorem 3.1). If A is a addition simple, then $K_0(A)$ is weakly unperforated (see Corollary 3.2). In fact, as we point out, $K_0(A)$ is weakly unperforated, but in the sense of Elliott [9], even when A is not simple. These results give new particular affirmative answers to a conjecture of B. Blackadar in [3]. For related results see [2], [4], [1], [20], [21], [7], [18], [14], [15] and [16]. The proofs of our above results rely on the new idea of using the fact that (SC) and cancellation are shape invariant (see Lemma 2.5) and they combine [11, Lemma 2.3, Remark 2.4 and Lemma 2.13] with techniques from [18].

The class of unital AH algebras which are proved in Theorem 3.1 to have cancellation and (SC) contains e.g. all the unital, real rank zero AH algebras $A = \varinjlim A_n'$, where the spectra of the trivial locally homogeneous algebras A_n' are uniformly bounded. This follows immediately from [22] and [11, Remark 1.4.6]. In our Theorem 3.3 we point out a new characterization of the real rank zero

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inductive limits of trivial locally homogeneous C^*-algebras whose spectra have uniformly bounded dimensions ($\leqq 3$) (see also [6] and [13]) which have been intensively studied by Elliott and Gong in their remarkable classification paper [11]. This characterization is an asymptotic condition in terms of the spectrum variation of the connecting homomorphisms and of the dimensions of the spectra of the building blocks in an appropriate inductive system of the C^*-algebra (in fact, it is a similar condition with the hypothesis of Theorem 3.1, but stronger in the unital case).

The results proved in this note can be used, in particular, to identify C^*-algebras which do not belong to "large" classes of AH algebras and also they are related to Elliott's project of the classification of the separable, nuclear C^*-algebras ([10]).

2. Preliminaries.

Let A be a unital C^*-algebra. We shall denote by $T(A)$ the set of all tracial states on A. We shall say that $p \in A$ is a projection if $p = p^* = p^2$. For two projections p, $q \in A$ we shall write $p \prec q$ if p is equivalent to a proper subprojection of q, i.e. $(\exists) v \in A$ such that $v^*v = p$ and $vv^* < q$. We shall say that $K_0(A)$ is weakly unperforated if whenever $n \cdot x > 0$ for some $x \in K_0(A)$ and some positive integer n, it follows that $x > 0$. For the definition and some properties of the spectrum variation $\mathrm{SPV}(\Phi)$ of a homomorphism Φ between two trivial homogeneous C^*-algebras (with metrizable spectra) see [11].

In what follows we shall need the following results and definitions:

THEOREM 2.1 (see [11, Lemma 2.3 and Remark 2.4]). *Let X be a connected finite* CW *complex with metric d, $F \subset C(X)$ be a finite set. For any integer N and $\varepsilon > 0$, there is $\delta > 0$ such that, if Φ: $C(X) \to PM_k(C(Y))P$ is a unital homomorphism with* $\mathrm{SPV}(\Phi) < \delta$, *where Y is a connected finite* CW *complex and $P \in M_k(C(Y))$ is a projection, then one of the following two statements is true:*

(a) $\mathrm{rank}(P) \geqq N$

(b) *There are $p_1, p_2, \ldots, p_n \in PM_k(C(Y))P$ with $\sum_i p_i = P$ and*

$$\|\Phi(f) - \sum_{i=1}^{n} f(x_i)p_i\| < \varepsilon \quad \text{for all} \quad f \in F$$

and furthermore, Φ is homotopic to Φ' defined by

$$\Phi'(f) = \sum_{i=1}^{n} f(x_i)p_i.$$

The following is also true: $\mathrm{SPV}(\Phi) < \dfrac{\delta}{\ell}$ *implies either* (a') $\mathrm{rank}(P) \geqq \ell N$ *or* (b) *as above,* THEOREM 2.2 (see [22, Theorem 2.5]). *Let A be a unital inductive limit of $(A_n, \Phi_{n,m})$ with $A_n = \oplus_{i=1}^{k_n} M_{[n,i]}(C(X_{n,i}))$, where $X_{n,i}$ are path connected metrizable spaces and $[n,i]$ are integers. If A is of real rank zero, then for any given n and $\varepsilon > 0$, there is an $m > n$ such that any partial homomorphism*

$$\Phi_{n,m}^{i,j}: M_{[n,i]}(C(X_{n,i})) \to M_{[m,j]}(C(X_{m,j}))$$

of $\Phi_{n,m}$ satisfies:

$$\mathrm{SPV}(\Phi_{n,m}^{i,j}) < \varepsilon.$$

The following lemma is a result of K. R. Goodearl ([16]) and N. C. Phillips ([19]):

LEMMA 2.3. *Let X be a compact space of dimension d and let p, q be projections in $C(X, M_n)$. If* $\operatorname{rank}(q(x)) - \operatorname{rank}(p(x)) \geqq \max\{(d-1)/2, 1\}$ *for all $x \in X$, then $p \prec q$.*

The following simple result is a slight improvement of Lemma 3.2 in [18] and its proof, which will be not given, follows an idea in [2]:

LEMMA 2.4. *Let $A = \varinjlim(A_n, \Phi_{n,m})$, where the C^*-algebras A_n are unital and have tracial states and the connecting homomorphisms $\Phi_{n,m}: A_n \to A_m$ are unital.*

Let p and q be projections in some A_n such that $\sigma(\tilde{p}) < \sigma(\tilde{q})$ for any $\sigma \in T(A)$, where $\tilde{p}$(resp. $\tilde{q}$) is the canonical image of p(resp. q) in the inductive limit $A = \varinjlim(A_n, \Phi_{n,m})$. Then there is $m > n$ such that:

$$\tau(\Phi_{n,m}(p)) < \tau(\Phi_{n,m}(q))$$

for any $\tau \in T(A_m)$.

DEFINITION (compare with [3]). A unital C^*-algebra A is said to have (SC) (*strict comparability*) if $T(A) \neq \phi$ and whenever p and q are projections in A with $\tau(p) < \tau(q)$ for any $\tau \in T(A)$ it follows that $p \prec q$.

The next proposition is a slight extension of a result in [18] and has a similar proof (which will be not given):

PROPOSITION 2.5 (see [18, Proposition 3.13 and its proof]). *Let $A = \varinjlim(A_n, \Phi_n)$ and $B = \varinjlim(B_n, \Psi_n)$, where A_n, B_n are arbitrary unital C^*-algebras and the connecting homomorphisms Φ_n, Ψ_n are unital.*

Suppose that there is an EP-*commutative diagram* (*see* [18, 2.3]) *with unital homomorphisms α_n and $\beta_n (n \geqq 1)$:*

$$\begin{array}{ccccccccc}
A_1 & \xrightarrow{\Phi_1} & A_2 & \xrightarrow{\Phi_2} & A_3 & \xrightarrow{\Phi_3} & A_4 & \xrightarrow{\Phi_4} & \cdots \\
 & {\scriptstyle\alpha_1}\searrow \ \nearrow{\scriptstyle\beta_1} & & {\scriptstyle\alpha_2}\searrow \ \nearrow{\scriptstyle\beta_2} & & {\scriptstyle\alpha_3}\searrow \ \nearrow{\scriptstyle\beta_3} & & & \\
 & B_1 & \xrightarrow{\Psi_1} & B_2 & \xrightarrow{\Psi_2} & B_3 & \xrightarrow{\Psi_3} & \cdots &
\end{array}$$

(*note that this happens if* e.g. *the above diagram is commutative at the level of homotopy*). *Then:*

a) $T(A) \neq \phi \Leftrightarrow T(B) \neq \phi$

b) *A has* (SC) $\Leftrightarrow$ *B has* (SC).

If furthermore the above diagram is a stably EP-*commutative diagram* (*that is, after taking the tensor product with* M_n *for any n, it is still an* EP-*commutative diagram*) *then:*

c) *A has cancellation* $\Leftrightarrow$ *B has cancellation.*

DEFINITION (compare with [7, 2.1.8]). Let $A = PM_k(C(X))P$ and $B = QM_\ell(C(Y))Q$ be homogeneous algebras, where X and Y are compact and connected and P and Q are projections. A *-homomorphism $\Phi: A \to B$ is said to be large if

a) $\Phi = 0$

or

b) $\Phi(P) = Q$ and $\operatorname{rank}(\Phi(P)) \geqq \max\{\dim(Y)/2, 1\} \operatorname{rank}(P)$.

3. The main results.

THEOREM 3.1. *Let*

$$A = \varinjlim(A_n, \Phi_{n,m})$$

where $A_n = \bigoplus_{i=1}^{s_n} P_{n,i}M_{[n,i]}(C(X_{n,i}))P_{n,i}$, $X_{n,i}$ *are finite connected* CW *complexes,* $P_{n,i} \in M_{[n,i]}(C(X_{n,i}))$ *are projections and* $\Phi_{n,m}$ *are unital. Let* $\delta(n,i)$ *be the largest strictly positive number* (*perhaps* $+\infty$) (*depending on* $X_{n,i}$ *only*) δ *such that* $B(x;a)$ *is contractible for any* $a < 2\delta$ *and any* $x \in X_{n,i}$, *where* $B(x;a)$ *denotes the closed ball in* $X_{n,i}$ *of center x and radius a. Suppose that for any given positive integer n there is* $m > n$ *such that any partial homomorphism*

$$\Phi_{n,m}^{i,j}: P_{n,i}M_{[n,i]}(C(X_{n,i}))P_{n,i} \to P_{m,j}M_{[m,j]}(C(X_{m,j}))P_{m,j}$$

of $\Phi_{n,m}$ *satisfies:*

$$\mathrm{SPV}(\Phi_{n,m}^{i,j}) < \frac{\delta(n_1,i)}{\max\{\dim(X_{m,j})/2, 1\}}$$

Then:

a) *A has* (SP) (*see Preliminaries for definition*).

b) *A has cancellation.*

PROOF. a) From the proof of [11, Lemma 2.3] it follows that (for any n and i) either $\delta(n,i)$ is a positive number δ as in [11, Lemma 2.3], corresponding to $X = X_{n,i}$, $F = \phi$, and $N = \varepsilon = 1$ or $\delta(n,i) = +\infty$. Let us introduce for any n and i the usual notation $A_n^i = P_{n,i}M_{[n,i]}(C(X_{n,i}))P_{n,i}$. Using now the hypothesis, it follows that there is $n_2 > n_1 = 1$ such that all the partial homomorphisms $\Phi^{i,j}_{1,n_2}\colon A^i_1 \to A^j_{n_2}$ satisfy:

$$\mathrm{SPV}(\Phi^{i,j}_{1,n_2}) < \frac{\delta(1,i)}{\max\{\dim(X_{n_2,j})/2, 1\}}$$

By Theorem 2.1 it follows that for any i,j either

$$\Phi^{i,j}_{1,n_2}\colon A^i_1 \to \Phi^{i,j}_{1,n_2}(1_{A^i_1})A^j_{n_2}\Phi^{i,j}_{1,n_2}(1_{A^i_1})$$

is large (see Preliminaries for definition) or $\Phi^{i,j}_{1,n_2}\colon A^i_1 \to \Phi^{i,j}_{1,n_2}(1_{A^i_1})A^j_{n_2}\Phi^{i,j}_{1,n_2}(1_{A^i_1})$ is homotopic to a unital $*$-homomorphism $A^i_1 \to \Phi^{i,j}_{1,n_2}(1_{A^i_1})A^j_{n_2}\Phi^{i,j}_{1,n_2}(1_{A^i_1})$ with finite dimensional image. Hence there is a diagram:

$$\begin{tikzcd} A_1 \arrow[rr, "\Phi_{1,n_2}"] \arrow[dr, "\alpha_1"'] & & A_{n_2} \\ & B_1 \arrow[ur, "\beta_1"'] & \end{tikzcd} \tag{1}$$

where:

i) α_1 and β_1 are unital $*$-homomorphisms

ii) $B_1 = \oplus_{j=1}^{t_1} B^j_1$ where for any j $B^j_1 = Q_j M_{m_j}(C(Z_j))Q_j$ with Z_j a finite connected CW complex and Q_j a projection in $M_{m_j}(C(Z_j))$

iii) any partial homomorphism $A^i_1 \to (\alpha_1)^{i,j}(1_{A^i_1})B^j_1(\alpha_1)^{i,j}(1_{A^i_1})$ induced by α_1 is either large of surjective with finite dimensional image.

iv) the above diagram (1) is commutative within homotopy.

Continuing in this way, we construct inductively a diagram of unital $*$-homomorphisms:

$$\begin{tikzcd} A_1 \arrow[rr, "\Phi_{n_1,n_2}"] \arrow[dr, "\alpha_1"'] & & A_{n_2} \arrow[rr, "\Phi_{n_2,n_3}"] \arrow[dr, "\alpha_2"'] & & A_{n_3} \arrow[r] & \cdots \arrow[r] & A_{n_m} \arrow[rr, "\Phi_{n_m,n_{m+1}}"] \arrow[dr, "\alpha_m"'] & & A_{n_{m+1}} \arrow[r] & \cdots \\ & B_1 \arrow[ur, "\beta_1"'] & & B_2 \arrow[ur, "\beta_2"'] & & & & B_m \arrow[ur, "\beta_m"'] & & \end{tikzcd}$$

which commutes within homotopy and each diagram:

$$\begin{tikzcd} A_{n_m} \arrow[rr, "\Phi_{n_m,n_{m+1}}"] \arrow[dr, "\alpha_1"'] & & A_{n_{m+1}} \\ & B_m \arrow[ur, "\beta_1"'] & \end{tikzcd}$$

has the above properties i)–iv). Using Proposition 2.5 and supposing that $n_m = m, m \geqq 1$ (to save the notation), we may assume that A is the C^*-inductive limit of the system:

$$A_1 \xrightarrow{\alpha_1} B_1 \xrightarrow{\beta_1} A_2 \xrightarrow{\alpha_2} B_2 \xrightarrow{\beta_2} A_3 \longrightarrow \ldots \longrightarrow A_n \xrightarrow{\alpha_n} B_n \xrightarrow{\beta_n} A_{n+1} \longrightarrow \ldots$$

Now let P and Q be projections in A with $\tau(P) < \tau(Q)$ for any $\tau \in T(A)$. By simple approximation arguments and changing the representatives in the inductive limit, since $A = \overline{\bigcup_{n=1}^{\infty} \mu_{n,\infty}(A_n)}$, we may suppose that $P = \mu_{1,\infty}(p)$, $Q = \mu_{1,\infty}(q)$ for some projections p and q in A_1. Here $\mu_{n,\infty}: A_n \to A = \varinjlim(A_m, \beta_m \circ \alpha_m)$ are the canonical *-homomorphisms.

Considering $A = \varinjlim(A_n, \beta_n \circ \alpha_n)$ and applying Lemma 2.4, we may suppose that:

$$\sigma(p) < \sigma(q) \quad \text{for any} \quad \sigma \in T(A_1) \tag{2}$$

Let $p = \oplus_{k=1}^{s_1} p_k$, $q = \oplus_{k=1}^{s_1} q_k \in A_1 = \oplus_{k=1}^{s_1} A_1^k$ where $A_1^k = P_{1,k} M_{[1,k]}(C(X_{1,k}))P_{1,k}$. By (2) it follows that:

$$\operatorname{rank}(p_k) < \operatorname{rank}(q_k), \quad 1 \leqq k \leqq s_1 \tag{3}$$

Let $(\alpha_1)^{i,j}: A_1^i \to (\alpha_1)^{i,j}(1_{A_1^i})B_1^j(\alpha_1)^{i,j}(1_{A_1^i})$ be the partial homomorphism induced by α_1. It can be shown (using [11, Lemma 2.13] and [18]) that there is an integer $k_{i,j} \geqq 0$ such that for any projection r in A_1^i we have:

$$\operatorname{rank}((\alpha_1)^{i,j}(r)) = k_{i,j} \cdot \operatorname{rank}(r) \tag{4}$$

We have to consider three cases for i,j fixed:

I) If $k_{i,j} = 0$, then $(\alpha_1)^{i,j}(p_i) = (\alpha_1)^{i,j}(q_i) = 0$

II) If $k_{i,j} > 0$ and $(\alpha_1)^{i,j}: A_1^i \to (\alpha_1)^{i,j}(1_{A_1^i})B_1^j(\alpha_1)^{i,j}(1_{A_1^i})$ is surjective with finite dimensional image, then using [11, Lemma 2.13] and [18] it is not difficult to see that $(\alpha_1)^{i,j}(p_i) \prec (\alpha_1)^{i,j}(q_i)$ (we use (3)).

III) If $k_{i,j} > 0$ and $(\alpha_1)^{i,j}: A_1^i \to (\alpha_1)^{i,j}(1_{A_1^i})B_1^j(\alpha_1)^{i,j}(1_{A_1^i})$ is large, then, using (4) and (3), we have:

$$\begin{aligned} \operatorname{rank}((\alpha_1)^{i,j}(q_i)) - \operatorname{rank}((\alpha_1)^{i,j}(p_i)) &= k_{i,j}(\operatorname{rank}(q_i) - \operatorname{rank}(p_i)) \\ &\geqq k_{i,j} \geqq \max\{\dim(Z_j)/2, 1\} \end{aligned}$$

By Lemma 2.3 this implies that:

$$(\alpha_1)^{i,j}(p_i) \prec (\alpha_1)^{i,j}(q_i) \quad \text{in} \quad B_1^j$$

In conclusion (using also the fact that A has (SC)), it follows that $P = \mu_{2,\infty}(\beta_1 \circ \alpha_1(p)) \prec Q = \mu_{2,\infty}(\beta_1 \circ \alpha_1(q))$.

b) Working as in the proof of a) we may suppose that every unital partial homomorphism induced by each $\Phi_{n,n+1}$ is either large (see Preliminaries) or homotopic to a homomorphism with finite dimensional image. Now the result follows using Lemma 2.5c) and stability results for vector bundles in [17].

COROLLARY 3.2. *Let A be as in Theorem* 3.1. *If in addition we assume that A is simple, then $K_0(A)$ is weakly unperforated.*

PROOF. Observe firstly that because A is in addition a simple C^*-algebra, for any two projections p and q in A we have:

$$p \prec q \quad \text{in} \quad A \Leftrightarrow \tau(p) < \tau(q) \quad \text{for any} \quad \tau \in T(A).$$

Since by Theorem 3.1 A has cancellation, in order to prove that $K_0(A)$ is weakly unperforated, it is enough to prove that if for some projections p and q in some matrix algebra B over A and some positive integer n we have:

$$(*) \qquad n \cdot p = \underbrace{p \oplus p \oplus \ldots \oplus p}_{n \text{ times}} \prec n \cdot q = \underbrace{q \oplus q \oplus \ldots \oplus q}_{n \text{ times}}$$

in $M_n(B)$, then:

$$(**) \qquad p \prec q \quad \text{in} \quad B$$

Let $\tau \in T(B)$. Then $\sigma = \tau \otimes \mathrm{tr}_n \in T(B \otimes M_n)$ if tr_n is the (unique) tracial state on M_n. By the above remark, $(*)$ implies that:

$$\sigma(n \cdot p) = \tau(p) < \sigma(n \cdot q) = \tau(q)$$

Since $\tau \in T(B)$ is arbitrary, by Theorem 3.1 a), it follows that $(**)$ is true.

REMARK. If one replaces the above weakly unperforated property in Corollary 3.2 by the one in the sense of Elliott ([9]), one can obtain the result without the restriction that A be simple. The proof goes using some of the arguments from the proof of Theorem 3.1.

The next result gives a new description of the AH algebras $A = \varinjlim A_n$ with real rank zero for which the dimensions of the spectra of the trivial locally homogeneous algebras A_n are $\leqq 3$ (see Theorem 3.3). Note that by a recent remarkable result proved by M. Dadarlat [6] and by Gong [13] it is known that this class of C^*-algebras coincides with the class of the AH algebras of real rank zero which are inductive limits of trivial locally homogeneous algebras with the dimensions of their spectra uniformly bounded. Our description is in terms of the spectrum variation of the connecting homomorphisms and of the dimensions of the spectra of the building blocks in an appropriate inductive system of the C^*-algebra (in fact is a similar condition with that in Theorem 3.1, but stronger in the unital case). In the simple case, by a remarkable result of Elliott and Gong in [11], this class of C^*-algebras is classified by the graded, scaled, ordered K_* group.

THEOREM 3.3. *Let A be a C^*-algebra. The following are equivalent:*
a) *A can be written*

$$A = \varinjlim(A_n, \Phi_{n,m}),$$

where $A_n = \oplus_{i=1}^{s_n} P_{n,i} M_{[n,i]}(C(X_{n,i}))P_{n,i}$, $X_{n,i}$ are finite connected CW *complexes,*

$[n, i]$, s_n are positive integers and $P_{n,i} \in M_{[n,i]}(C(X_{n,i}))$ are projections such that:
For any given $\varepsilon > 0$ and any given positive integer n, there is $m_0 > n$ such that any partial homomorphism

$$\Phi^{i,j}_{n,m}\colon P_{n,i}M_{[n,i]}(C(X_{n,i}))P_{n,i} \to P_{m,j}M_{[m,j]}(C(X_{m,j}))P_{m,j}$$

of $\Phi_{n,m}\colon A_n \to A_m$ with $m \geqq m_0$ satisfies

$$\mathrm{SPV}(\Phi^{i,j}_{n,m}) < \frac{\varepsilon}{\dim(X_{m,j}) + 1}$$

b) *A has real rank zero and $A = \varinjlim(A'_n, \Psi_{n,m})$ where*

$$A'_n = \oplus_{i=1}^{t_n} Q_{n,i}M_{\{n,i\}}(C(Y_{n,i}))Q_{n,i}$$

$Y_{n,i}$ are connected CW complexes of dimension $\leqq 3$, $H^3(Y_{n,i})$ are finite, $\{n, i\}$, t_n are positive integers and $Q_{n,i} \in M_{\{n,i\}}(C(Y_{n,i}))$ are projections.

PROOF. a) $\Rightarrow$ b). As in the proof of [11, Corollary 2.25] it follows that A has real rank zero. Now, the proof goes as in the proof of Theorem 3.2 in [6] (the case when $\sup_{n,i}(\dim(X_{n,i})) < +\infty$) using the hypothesis and Theorem 2.1 (i.e. [11, Lemma 2.3 and Remark 2.4]) instead of the fact that the real rank of A is zero, of Theorem 2.2 (i.e. [22, Theorem 2.5]) and [11, Remark 1.4.6] and of [11, Lemma 2.3] (to get m-large *-homomorphisms in the sense of [7] or *-homomorphisms with finite dimensional image) and observing that the appropriate variants of [11, Theorem 2.29 and Remark 2.30] hold if we replace the conditions that A has real rank zero and $\sup_{n,i}(\dim(X_{n,i})) < +\infty$ by our hypothesis.

b) $\Rightarrow$ a). Let $\varepsilon > 0$ and let a positive integer n be fixed. Then, by Theorem 2.2 and [11, Remark 1.4.6], there is $m_0 > n$ such that for any partial homomorphism $\Psi^{i,j}_{n,m}$ we have:

$$\mathrm{SPV}(\Psi^{i,j}_{n,m}) < \frac{\varepsilon}{4} \quad \text{for any} \quad m \geqq m_0.$$

On the other hand, the hypothesis obviously implies that:

$$\frac{\varepsilon}{4} \leqq \frac{\varepsilon}{\dim(Y_{m,j}) + 1}.$$

Hence:

$$\mathrm{SPV}(\Psi^{i,j}_{n,m}) < \frac{\varepsilon}{\dim(Y_{m,j}) + 1}$$

for all the partial homomorphisms $\Psi^{i,j}_{n,m}$ with $m \geqq m_0$.

REMARK. We would like to point out that until now most of the known results about inductive limit C^*-algebras (concerning real rank, stable rank, exponential rank, cancellation, (SC), classification etc.) have been obtained for C^*-algebras defined by inductive systems with slow dimension growth in one sense or another (exceptions are above, in [12], [18] and eventually in other few places). Note also that the system satisfying the condition a) in the above theorem generally has NO slow dimension growth in any sense, in contrast with the system in Theorem 3.3 b) which defines the same algebra.

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