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PRIMITIVE IDEMPOTENTS OF THE GROUP ALGEBRA CSL(3, q)

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Introduction.

Let GL(3, q) denote the group of all nonsingular 3×3 matrices over the Galois field GF(q) of q elements ($q = p^s$, p is a prime, $s \geqq 1$ is an integer) and C be the field of all complex numbers. Drobotenko [3] obtained the primitive idempotents of the group algebra CGL(3, q). For this, he used the following properties:

PROPERTY 1. *Let G be a finite group of order n and let K be an algebraically closed field with characteristic not dividing n. If ξ is an irreducible KG-character affording the orthogonal central idempotent e_ξ of KG, then*

$$e_\xi = \xi(1)n^{-1} \sum_{g \in G} \xi(g^{-1})g,$$

where $\xi(1)$ is the degree of ξ.

PROPERTY 2. *Let H be a subgroup of G and let ψ be a KH-character of degree $1, \eta$ an irreducible KG-character. Assume that the multiplicity of η in the induced KG-character ψ^G is 1. If η and ψ afford the orthogonal central idempotents e_η and e_ψ respectively, then $e_\eta e_\psi$ is a primitive idempotent of KG which corresponds to η.*

PROOF. Let M be a KH-module and N be an irreducible KG-module which afford the characters ψ and η respectively. From Wedderburn theorem [2, Theorem 2.9], it is easy to see that $KHe_\psi = M$ and $e_\eta M^G = N$. By definition of the induced KG-module M^G, we obtain $M^G = KGe_\psi$. Then we have $N = KGe_\psi e_\eta$. Moreover, it is well known that e is a primitive idempotent of KG if and only if KGe is an irreducible KG-module.

REMARK. Using these properties, all irreducible K-representations of G are obtained.

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We have used these properties in [4] to obtain the primitive idempotents of CSL$(3, p)$. In this paper we will use again these properties and the some results of [3] to obtain the primitive idempotents of CSL$(3, q)$ which correspond to the irreducible CSL$(3, q)$-characters, where SL$(3, q)$ is the group of 3×3 matrices over GF(q) with determinant unity.

Method and Results.

If ρ is a generator of the multiplicative group GF$(q)^\times$ and if d is the greatest common divisor of $q-1$ and 3, we write $\langle p \rangle = \mathrm{GF}(q)^\times$ and $d = (q-1,3)$ respectively. Let $\mathrm{K} = \{\alpha \in \mathrm{GF}(q)^\times \mid \alpha \equiv p^u;\ u \equiv 0 (\mathrm{mod}\, 3\}$. If $d = 1$, we have $\mathrm{GF}(q)^\times = \mathrm{K}$ and if $d = 3$, then

$$\mathrm{GF}(q)^\times = \mathrm{K} \cup \rho\mathrm{K} \cup \rho^2\mathrm{K} \quad \text{(disjoint)}.$$

Let θ be a primitive element of the field GF(q). Consider the above elements of $S = \mathrm{SL}(3, q)$:

$$a_i = \begin{bmatrix} 1 & 0 & \theta^{s-i} \\ & 1 & 0 \\ & & 1 \end{bmatrix}, b_i = \begin{bmatrix} 1 & \theta^{s-i} & 0 \\ & 1 & 0 \\ & & 1 \end{bmatrix}, c = \begin{bmatrix} 1 & 0 & 0 \\ & 1 & \theta^{s-i} \\ & & 1 \end{bmatrix}$$

$$(a_i^p = b_i^p = c_i^p = I;\ i = 1, \ldots, s)$$

It follows directly that S has the following subgroups:

$A = \langle a_1 \rangle \times \ldots \times \langle a_s \rangle$; $B = \langle b_1 \rangle \times \ldots \times \langle b_s \rangle$; $C = \langle c_1 \rangle \times \ldots \times \langle c_s \rangle$;
$D = A \times B$; $H = DC$;

$$|A| = |B| = |C| = q;\ |D| = q^2;\ |H| = q^3.$$

The irreducible CH-characters are given by table 1 [3]. It is easy to see that there exist the following relations between the conjugacy classes of S which have already been determined by Dickson [1] and the conjugacy classes of H:

The conjugacy classes of types $\Delta_3, \Delta_4, \Delta_5$ of H, for $d = 1$ or 3, are included in the conjugacy class of canonical representative $\begin{bmatrix} 1 & 1 & 0 \\ & 1 & 0 \\ & & 1 \end{bmatrix}$ of S.

The conjugacy classes of type Δ_5 of H, for $d = 1$, are included in the conjugacy class of canonical representative $\begin{bmatrix} 1 & 1 & 0 \\ & 1 & 1 \\ & & 1 \end{bmatrix}$ of S and for $d = 3$, we have the following proposition:

Proposition 1. *For $d = 3$, $(q-1)^2/3$ conjugacy classes of type Δ_5 of H are included in the conjugacy classes of canonical representatives*

TABLE 1
Irreducible CH-characters

$k = k_1 + k_2\theta + \ldots + k_s\theta^{s-1}$, $t = t_1 + t_2\theta + \ldots + t_s\theta^{s-1}$; $k_i, t_i, m_i, n_i = 0, \ldots, p-1$; $i = 1, \ldots s$; $(k_1, \ldots, k_s) \neq (0, \ldots 0)$, $(t_1, \ldots t_s) \neq (0, \ldots, 0)$, $(h_1, \ldots, h_s) \neq (0, \ldots, 0)$, $\varepsilon \neq 1$; $\varepsilon^p = 1$

Conjugacy class	canonical representative	number of conjugacy classes	Number of elements in the conj. class	$\phi^H_{h_1,\ldots,h_s}$	$\psi_{m_1,\ldots,m_s;n_1,\ldots n_s}$
Δ_1	$\begin{bmatrix}1 & 0 & 0\\ & 1 & 0\\ & & 1\end{bmatrix}$	1	1	q	1
Δ_2	$\begin{bmatrix}1 & 0 & k\\ & 1 & 0\\ & & 1\end{bmatrix}$	$q-1$	1	$q\varepsilon^{k_1h_1+\ldots+k_sh_s}$	1
Δ_3	$\begin{bmatrix}1 & k & 0\\ & 1 & 0\\ & & 1\end{bmatrix}$	$q-1$	q	0	$\varepsilon^{k_1m_1+\ldots+k_sm_s}$
Δ_4	$\begin{bmatrix}1 & 0 & 0\\ & 1 & t\\ & & 1\end{bmatrix}$	$q-1$	q	0	$\varepsilon^{t_1n_1+\ldots+t_sn_s}$
Δ_5	$\begin{bmatrix}1 & k & 0\\ & 1 & t\\ & & 1\end{bmatrix}$	$(q-1)^2$	q	0	$\varepsilon^{k_1m_1+\ldots+k_sm_s+t_1n_1+\ldots+t_sn_s}$

$$\begin{bmatrix}1 & 1 & 0\\ & 1 & 1\\ & & 1\end{bmatrix}, \begin{bmatrix}1 & 1 & 0\\ & 1 & \rho\\ & & 1\end{bmatrix}, \begin{bmatrix}1 & 1 & 0\\ & 1 & \rho^2\\ & & 1\end{bmatrix}$$

of S if and only if the following conditions hold respectively:

X) $k \equiv t \pmod{\mathsf{K}}$

Y) $(k, t) \in \mathsf{K} \times \rho\mathsf{K}$ *or* $(k, t) \in \rho\mathsf{K} \times \rho^2\mathsf{K}$ *or* $(k, t) \in \rho^2\mathsf{K} \times \mathsf{K}$

Z) $(k, t) \in \mathsf{K} \times \rho^2\mathsf{K}$ *or* $(k, t) \in \rho\mathsf{K} \times \mathsf{K}$ *or* $(k, t) \in \rho^2\mathsf{K} \times \rho\mathsf{K}$

Let $G = \mathrm{GL}(3, q)$. The irreducible CG-characters are as follows [6]:

$$\xi^{(j)}_z;\ j = 1, \ldots, 8;\ z = 1, \ldots, h_j;\ h_1 = h_2 = h_4 = q - 1,$$

(1) $$h_3 = h_5 = (q-1)(q-2), h_6 = (q-1)(q-2)(q-3)/6,$$

$$h_7 = q(q-1)^2/2, h_8 = q(q^2-1)/3;$$

where j denotes the type of the character and z denotes the number of the characters of the type j.

The irreducible CS-characters are as follows [5]:

$$\eta_r^{(j)}; j = 1,\dots,11; r = 1,\dots,f_j; f_1 = f_2 = f_4 = 1, f_3 = f_5 = q-2,$$
$$(2) \qquad f_6 = [(q-1)(q-4) + 3 - d]/6, f_7 = q(q-1)/2,$$
$$f_8 = (q^2 + q + 1 - d)/3, f_9 = f_{10} = f_{11} = (d-1)3/2;$$

where $d = (q-1, 3) = 1$ or 3 and j denotes the type of the character, r denotes the number of the characters of the type j.

Let $\xi_{z|S}^{(j)}$ be the restriction of $\xi_z^{(j)}$ to S. The relations between (1) and (2) are as follows:

$$\text{For } d = 1,\ \xi_{z_r|S}^{(j)} = \eta_r^{(j)}; j = 1,\dots,8; z = 1,\dots,h_j; r = 1,\dots,f_j$$
$$\text{For } d = 3,\ \xi_{z_r|S}^{(j)} = \eta_r^{(j)}; j = 1,\dots,8; z = 1,\dots,h_j; r = 1,\dots,f_j,$$
$$(3) \qquad \sum_{r=0}^{2} \eta_r^{(9)} = \xi_{z_1|S}^{(6)}, \sum_{r=0}^{2} \eta_r^{(10)} = \xi_{z_2|S}^{(8)}, \sum_{r=0}^{2} \eta_r^{(11)} = \xi_{z_3|S}^{(8)},$$

where $\xi_{z_r}^{(j)}$ are conveniently chosen CG-characters. Using the character tables which are given in [5] and [6] we can chose easily the character $\xi_{z_r}^{(j)}$.

Moreover, from the table of the irreducible CG-characters which is given in [6] and the table 1, it is easy to obtain the multiplicity of $\xi_z^{(j)}$ in $\psi_{m_1,\dots,m_s;n_1,\dots,n_s}^G$.

Then we have [3]:

$$\psi_{m_1,\dots,m_s;0,\dots 0}^G = \psi_{0,\dots 0;n_1,\dots n_s}^G = \sum_{j=2}^{4} \sum_{z=1}^{h_j} \xi_z^{(j)} + 2 \sum_{z=1}^{h_5} \xi_z^{(5)} +$$
$$(4) \qquad + 3 \sum_{z=1}^{h_6} \xi_z^{(6)} + \sum_{z=1}^{h_7} \xi_z^{(7)},$$
$$\psi_{m_1,\dots,m_s;n_1,\dots n_s}^G = \sum_{j=4}^{8} \sum_{z=1}^{h_j} \xi_z^{(j)}$$

$(m_i, n_i = 0,\dots p-1; i = 1,\dots,s; (m_1,\dots,m_s) \neq (0,\dots,0); (n_1,\dots,n_s) \neq (0,\dots,0))$

Let $\eta_{r|H}^{(j)} = \eta^{(j)}$. The values $\eta^{(j)}$ are given by table 2. Let

$$(\psi_{m_1,\dots,m_s;n_1,\dots,n_s}^S; \eta_r^{(j)})$$

denote the multiplicity of $\eta_r^{(j)}$ in $\psi_{m_1,\dots,m_s;n_1,\dots,n_s}^S$. By the tables 1 and 2, the proposition 1 and the Frobenius theorem, it is easy to see that, for every $r \in \{0, 1, 2\}$, $(\psi_{m_1,\dots,m_s;0,\dots,0}^S; \eta_r^{(j)})$ (and $(\psi_{0,\dots,0;n_1,\dots,n_s}^S; \eta_r^{(j)})$) is constant. Moreover, from (3) and (4), we have:

TABLE 2

Values of the irreducible CSL$(3, q)$-characters on H

Con. class	Canon. repres.	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$	$\eta^{(4)}$	$\eta^{(5)}$	$\eta^{(6)}$	$\eta^{(7)}$	$\eta^{(8)}$	$\eta^{(9)}$	$\eta^{(10)}$	$\eta^{(11)}$
Δ_1	$\begin{bmatrix}1&0&0\\&1&0\\&&1\end{bmatrix}$	1	$q(q+1)$	q^2+q+1	q^3	$q(q^2+q+1)$	$(q+1)\times$ (q^2+q+1)	$(q-1)\times$ (q^2+q+1)	$(q-1)\times$ $(q+1)$	$(q+1)\times$ $(q^2+q+1)/3$	$(q-1)^2\times$ $(q+1)/3$	$(q-1)^2\times$ $(q+1/3$
Δ_2	$\begin{bmatrix}1&0&k\\&1&0\\&&1\end{bmatrix}$	1	q	$q+1$	0	q	$2q+1$	-1	$-(q-1)$	$(2q+1)/3$	$-(q-1)/3$	$-(q-1)/3$
Δ_3	$\begin{bmatrix}1&k&0\\&1&0\\&&1\end{bmatrix}$	1	q	$q+1$	0	q	$2q+1$	-1	$-(q-1)$	$(2q+1)/3$	$-(q-1)/3$	$-(q-1)/3$
Δ_4	$\begin{bmatrix}1&0&0\\&1&t\\&&1\end{bmatrix}$	1	q	$q+1$	0	q	$2q+1$	-1	$-(q-1)$	$(2q+1)/3$	$-(q-1)/3$	$-(q-1)/3$
Δ_5	$\begin{bmatrix}1&k&0\\&1&t\\&&1\end{bmatrix}$	1	0	1	0	0	1	-1	1	$q\delta_{rh}-$ $-(q-1)/3$	$q\delta_{rh}-$ $-(q-1)/3$	$q\delta_{rh}-$ $-(q-1)/3$

$r, h = 0, 1, 2;\ \delta_{rh} = \begin{cases} 1, & r = h \\ 0, & r \quad h \end{cases}$

$$(\psi^S_{m_1,\ldots,m_s;0,\ldots,0};\ \sum_{r=0}^{2} \eta_r^{(9)}) = (\psi^S_{0,\ldots,0;n_1,\ldots,n_s};\ \sum_{r=0}^{2} \eta_r^{(9)}) = 3.$$

It follows directly that:

$$(5)\qquad (\psi^S_{m_1,\ldots,m_s;0,\ldots,0};\ \eta_r^{(9)}) = (\psi^S_{0,\ldots,0;n_1,\ldots,n_s};\ \eta_r^{(9)}) = 1 \qquad (r = 0, 1, 2)$$

Finally, by the tables 1 and 2, (5) and the Frobenius theorem, we obtain the following relation:

$$(6)\qquad \psi^S_{m_1,\ldots,m_s;0,\ldots,0} = \psi^S_{0,\ldots,0;n_1,\ldots,n_s} = \sum_{j=2}^{4}\sum_{r=1}^{f_j} \eta_r^{(j)} + 2\sum_{r=1}^{f_5} \eta_r^{(5)} + {} + 3\sum_{r=1}^{f_6} \eta_r^{(6)} + \sum_{r=1}^{f_7} \eta_r^{(7)} + \left(\frac{d-1}{2}\right)\sum_{r=0}^{2} \eta_r^{(9)}$$

$(m_i, n_i = 0, \ldots, p-1; i = 1, \ldots, s; (m_1, \ldots, \mathrm{m}_s) \neq (0, \ldots, 0); (n_1, \ldots, n_s) \neq (0, \ldots, 0);$

$d = 1, 3)$

If (k, t) satisfies the condition X (or Y or Z), we write $(k, t) \in \mathrm{X}$ (or $(k, t) \in \mathrm{Y}$ or $(k, t) \in \mathrm{Z}$). It follows from the tables 1 and 2, (4) and the Frobenius theorem that:

$$(\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s};\ \eta_r^{(j)}) = q^{-3}\Bigg[(q^3 - q)/3 + (q\delta_{r0} - (q-1)/3)q \sum_{(k,t)\in \mathrm{X}} \varepsilon^\alpha + {} + (q\delta_{r1} - (q-1)/3)q \sum_{(k,t)\in \mathrm{Y}} \varepsilon^\alpha + (qd_{r2} - (q-1)/3)q \sum_{(k,t)\in \mathrm{Z}} \varepsilon^\alpha\Bigg] =$$

$$(7)\qquad = 1 \text{ or } 0.$$

$(j = 9, 10, 11; r = 0, 1, 2; \delta_{rh} = \begin{cases} 1; r = h \\ 0; r \neq h \end{cases}, \varepsilon \neq 1; \varepsilon^p = 1, k = k_1 + k_2\theta + \ldots + k_s\theta^{s-1},$

$t = t_1 + t_2\theta + \ldots + t_s\theta^{s-1}; \alpha = k_1m_1 + \ldots + k_sm_s + t_1n_1 + \ldots + t_sn_s; \mathrm{m}_i,$

$n_i = 0, \ldots, p-1; i = 1, \ldots, s; (m_1, \ldots, m_s) \neq (0, \ldots, 0), (n_1, \ldots, n_s) \neq (0, \ldots 0),$

$(k_1, \ldots, k_s) \neq (0, \ldots, 0), (t_1, \ldots, t_s) \neq (0, \ldots, 0))$

Using (7), we then obtain the following result, which follows from the solution of a simple system of linear equations:

PROPOSITION 2. *Let* $x = \sum_{(k,t)\in \mathrm{X}} \varepsilon^\alpha$, $y = \sum_{(k,t)\in \mathrm{Y}} \varepsilon^\alpha$, $z = \sum_{(k,t)\in \mathrm{Z}} \varepsilon^\alpha$ *in* (7). *One of the* x, y, z *is equal to* $(2q + 1)/3$ *and each of the others is equal to* $(1 - q)/3$.

We now prove the following basic result:

PROPOSITION 3. *For* $j = 9, 10, 11,$

$$(\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s}; \eta^{(j)}_r) = 1; \quad \begin{array}{l} (m_i, n_i) \in \mathrm{X},\ r = 0; \\ (m_i, n_i) \in \mathrm{Z},\ r = 1; \\ (m_i, n_i) \in \mathrm{Y},\ r = 2. \end{array}$$

$$(m_i, n_i = 0, \ldots, p-1; m_i = 0 \Leftrightarrow n_i = 0; i = 1, \ldots, s; (m_1, \ldots, m_s) \neq (0, \ldots 0), \\ (n_1, \ldots, n_s) \neq (0, \ldots 0))$$

PROOF. Set $n_i = m_i$ $(i = 1, \ldots, s)$ in (7). Then we have:

$$(\psi^S_{m_1,\ldots,m_s;m_1,\ldots,m_s}; \eta^{(j)}_0) = q^{-3}[(q^3 - q)/3 + ax + by + bz] = 1 \text{ or } 0;$$
$$a = (q - (q-1)/3)q,\ b = -(q-1)q/3,\ j = 9, 10, 11.$$

Since the elements k_i, t_i $(i = 1, \ldots, s)$ obtained for $(k, t) \in \mathrm{Y}$ are the same for $(t, k) \in \mathrm{Z}$, proposition 2 implies that $x = (2q+1)/3$ and $y = z = (1-q)/3$. Hence, we have:

$$(8)\quad (\psi^S_{m_1,\ldots,m_s;m_1,\ldots,m_s}; \eta^{(j)}_0) = q^{-3}[(q^3 - q)/3 + ax + by + bz] = 1; j = 9, 10, 11.$$

For all elements m of $\{0, \ldots, p-1\} \cap \rho\mathrm{K}$, we have the following relations:

$$(k, t) \in \mathrm{X} \Rightarrow (k, mt) \in \mathrm{Y},$$
$$(k, t) \in \mathrm{Y} \Rightarrow (k, mt) \in \mathrm{Z},$$
$$(k, t) \in \mathrm{Z} \Rightarrow (k, mt) \in \mathrm{X}.$$

Putting $\beta = m_1k_1 + \ldots + m_sk_s + mm_1t_1 + \ldots + mm_st_s$ and

$$\beta' = m_1k_1 + \ldots m_sk_s + m_1t'_1 + \ldots m_st'_s;\ t'_i = mt_i;\ i = 1, \ldots, s,$$
$$t' = t'_1 + t'_2\theta + \ldots + t'_s\theta^{s-1}.$$

We obtain:

$$\sum_{(k,t)\in\mathrm{X}} \varepsilon^\beta = \sum_{(k,t')\in\mathrm{Y}} \varepsilon^{\beta'} = y,\ \sum_{(k,t)\in\mathrm{Y}} \varepsilon^\beta = \sum_{(k,t')\in\mathrm{Z}} \varepsilon^\beta = z,\ \sum_{(k,t)\in\mathrm{Z}} \varepsilon^\beta = \sum_{(k,t')\in\mathrm{X}} \varepsilon^\beta = x.$$

After putting $mm_i = n_i$ $(i = 1, \ldots, s)$, we obtain:

$$(9)\quad \begin{array}{c} (\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s}; \eta^{(j)}_2) = q^{-3}[(q^3 - q)/3 + by + bz + ax] = 1 \\ (m_i = 0 \Leftrightarrow n_i = 0; (m_i, n_i) \in \mathrm{Y};\ i = 1, \ldots, s, j = 9, 10, 11) \end{array}$$

We note that we pass from (8) to (9) with the permutation (xzy) afforded by $\sum \varepsilon^\beta$. Moreover, for all elements m of $\{0, \ldots, p-1\} \cap p\mathrm{K}$, we have:

$$(k, t) \in \mathrm{X} \Rightarrow (mk, t) \in \mathrm{Z},\ (k, t) \in \mathrm{Y} \Rightarrow (mk, t) \in \mathrm{X}, (k, t) \in \mathrm{Z} \Rightarrow (mk, t) \in \mathrm{Y}$$

It follows that $\sum \varepsilon^{\gamma}$; $\gamma = mm_1k_1 + \ldots + mm_sk_s + m_1t_1 + \ldots + m_st_s$ affords the permutation (xyz). Hence, we obtain:

$$(10)\qquad \begin{gathered}(\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s};\ \eta^{(j)}_1) = 1\\ (m_i = 0 \Leftrightarrow n_i = 0;\ (m_i, n_i) \in \mathrm{Z};\ i = 1, \ldots, s, j = 9, 10, 11)\end{gathered}$$

For all elements m of $\{0, \ldots, p-1\} \cap \rho^2\mathrm{K}$, we have:

$$(k, t) \in \mathrm{X} \Rightarrow (mk, t) \in \mathrm{Y} \text{ and } (k, mt) \in \mathrm{Z}$$

$$(k, t) \in \mathrm{Y} \Rightarrow (mk, t) \in \mathrm{Z} \text{ and } (k, mt) \in \mathrm{X}$$

$$(k, t) \in \mathrm{Z} \Rightarrow (mk, t) \in \mathrm{X} \text{ and } (k, mt) \in \mathrm{Y}$$

It follows directly that $\sum \varepsilon^{\gamma}$ and $\sum \varepsilon^{\beta}$ afford the permutation (xzy) and (xyz) respectively. Hence, we obtain:

$$(11)\qquad \begin{gathered}(\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s};\ \eta^{(j)}_2) = 1;\ (m_i, n_i) \in \mathrm{Y},\\ (\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s};\ \eta^{(j)}_1) = 1;\ (m_i, n_i) \in \mathrm{Z},\\ (m_i = 0 \Leftrightarrow n_i = 0;\ i = 1, \ldots, s;\ j = 9, 10, 11)\end{gathered}$$

In the same way, for all elements m of $\{0, \ldots, p^{-1}\} \cap \mathrm{K}, \sum \varepsilon^{\gamma}$ afford the identity permutation. Hence, we obtain:

$$(12)\qquad \begin{gathered}(\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s};\ \eta^{(j)}_0) = 1\\ (m_i = 0 \Leftrightarrow n_i = 0;\ (m_i, n_i) \in \mathrm{X};\ i = 1, \ldots, s;\ j = 9, 10, 11)\end{gathered}$$

This completes the proof of proposition.

Finally, by the tables 1 and 2, (9), (10), (11), (12) and the Frobenius theorem, we obtain the following relations:

$$(13)\qquad \begin{gathered}\psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s} = \sum_{j=4}^{8} \sum_{r=1}^{h_j} \eta^{(j)}_r + \left(\frac{d-1}{2}\right) \sum_{j=9}^{11} \eta^{(j)}_0;\ (m_i, n_i) \in \mathrm{X},\\ \psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s} = \sum_{j=4}^{8} \sum_{r=1}^{h_j} \eta^{(j)}_r + \left(\frac{d-1}{2}\right) \sum_{j=9}^{11} \eta^{(j)}_1;\ (m_i, n_i) \in \mathrm{Z},\\ \psi^S_{m_1,\ldots,m_s;n_1,\ldots,n_s} = \sum_{j=4}^{8} \sum_{r=1}^{h_j} \eta^{(j)}_r + \left(\frac{d-1}{2}\right) \sum_{j=9}^{11} \eta^{(j)}_2;\ (m_i\ n_i) \in \mathrm{Y},\end{gathered}$$

$$(m_i, n_i = 0, \ldots, p-1;\ m_i = 0 \Leftrightarrow n_i = 0;\ i = 1, \ldots, s;\ (m_1, \ldots, m_s) \neq (0, \ldots, 0),\ (n_1, \ldots, n_s) \neq (0, \ldots, 0))$$

To summarize, using (3), (4), (6), (13) and property 2, we have established the following theorem:

THEOREM. *If* $e_r^{(j)}$ *and* $e_{m_1,\dots,m_s;n_1,\dots,n_s}$ *are the orthogonal central idempotents afforded by the irreducible* CSL(3, q)*-character* $\eta_r^{(j)}$ *and the irreducible* CH*-character* $\psi_{m_1,\dots,m_s;n_1,\dots,n_s}$ *respectively, then the primitive idempotents of the group algebra* CSL(3, q) *which correspond to* $\eta_r^{(j)}$ *are as follows:*

$$For\ j = 2,3,4,9;\ e_r^{(j)} e_{m_1,\dots,m_s;0,\dots,0},\ e_r^{(j)} e_{0,\dots,0;n_1,\dots,n_s}$$

$$For\ j = 4,5,6,7,8;\ e_r^{(j)} e_{m_1,\dots,m_s;n_1,\dots,n_s}$$

$$(m_i, n_i = 0,\dots,p-1;\ i = 1,\dots,s;\ (m_1,\dots,m_s) \neq (0,\dots,0),\ (n_1,\dots,n_s) \neq (0,\dots,0))$$

$$For\ j = 9,10,11;\ e_0^{(j)} e_{m_1,\dots,m_s;n_1,\dots,n_s};\ (m_i, n_i) \in \mathrm{X},$$

$$e_1^{(j)} e_{m_1,\dots,m_s;n_1,\dots,n_s};\ (m_i, n_i) \in \mathrm{Z},$$

$$e_2^{(j)} e_{m_1,\dots,m_s;n_1,\dots,n_s};\ (m_i, n_i) \in \mathrm{Y}.$$

$$(m_i, n_i = 0,\dots,p-1;\ m_i = 0 \Leftrightarrow n_i = 0;\ i = 1,\dots,s;\ (m_1,\dots,m_s) \neq (0,\dots,0),\ (n_1,\dots,n_s) \neq (0,\dots,0))$$

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