MATH. SCAND. 51 (1982), 142–150

A NOTE ON THE CHARACTERS OF THE PROJECTIVE MODULES FOR THE INFINITESIMAL SUBGROUPS OF A SEMISIMPLE ALGEBRAIC GROUP

STEPHEN DONKIN

Let G be an affine, connected, semisimple, simply connected algebraic group over an algebraically closed field k of characteristic $p \neq 0$. Let T be a maximal torus of G and, for a positive integer n, let $\boldsymbol{u}_n$ be the nth hyperalgebra of G (introduced in [5]). The representations of $\boldsymbol{u}_n$ correspond naturally to the rational representations of the infinitesimal subgroup G_n of G (see § 2 of [8]). In a recent paper, [7], Jantzen considers a category of modules for $\boldsymbol{u}_n$ and T simultaneously — the category of $\boldsymbol{u}_n$-T-modules. (We assume of the reader familiarity with the basic theory of $\boldsymbol{u}_n$-T-modules, as developed in [7].) In particular he shows (Satz of § 4 of [7]) that each projective indecomposable $\boldsymbol{u}_n$-module has naturally the structure of a $\boldsymbol{u}_n$-T-module. It is our purpose here to show that the characters of these modules are characters of rational G-modules. When p is large each projective indecomposable $\boldsymbol{u}_n$-module may be given the structure of a rational G-module (see [1] and § 4 of [8]), so our result may be seen as grounds for the hope that this is the case in all characteristics.

The result is obtained as a corollary to the following.

THEOREM. *Let M be a finite dimensional rational G-module with a decomposition $M = Q \oplus R$ as a $\boldsymbol{u}_n$-T-module. Suppose that the $\boldsymbol{u}_n$-socle of Q is simple and is a G-submodule of M. Then the character* $\operatorname{ch} Q$ *of Q is a proper character.*

By a proper character we mean the character of a rational G module.

In section 1 we deal with some general properties of proper characters. We begin section 2 with a construction of a group G^* from the decomposition $M = Q \oplus R$ of the theorem. The group is very similar to that considered by Cline in § 1 of [2]. Indeed, it is clear that such a construction is possible for a much

Received October 7, 1980.

wider class of algebraic groups and representations of (not necessarily reduced) closed normal subgroups than those considered here. However, we decided to sacrifice this generality in favour of concreteness and brevity. We conclude with a proof of the theorem and of its corollary.

The author was a visitor at The University of Turku, Finland whilst this paper was written. I wish to thank the Department of Mathematics for hospitality and The Royal Society and The Science Research Council of Great Britain, The Academy of Finland and The University of Turku for financial support.

1. Characters.

Let k be an algebraically closed field. All varieties considered here (in particular algebraic groups) are affine varieties over k. For an algebraic group G we denote by $\mathscr{M}_G$ the category of finite dimensional rational G-modules.

Let T be a torus. We denote by $X(T)$ the set of one dimensional rational representations considered as an additive abelian group. We write $\mathscr{G}(T) = \mathbf{Z}X(T)$, the integral group ring of $X(T)$; $\mathscr{G}(T)$ has a canonical basis $\{e(\lambda) : \lambda \in X(T)\}$ with multiplication satisfying $e(\lambda)e(\lambda') = e(\lambda+\lambda')$. Any rational T-module V is completely reducible; we have

$$V = \sum_{\lambda \in X(T)}^{\oplus} V^{\lambda} ,$$

where

$$V^{\lambda} = \{v \in V : \ tv = \lambda(t)v \quad \text{for all } t \in T\} .$$

For $V \in \mathscr{M}_G$ we define the character ch V of V by

$$\operatorname{ch} V = \sum_{\lambda \in X(T)} (\dim_k V^{\lambda}) e(\lambda) ,$$

if V is not zero and define the character of the zero module to be the zero element of $\mathscr{G}(T)$.

If

$$0 \to V' \to V \to V'' \to 0$$

is a short exact sequence of modules in $\mathscr{M}_T$, we have

$$\operatorname{ch} V = \operatorname{ch} V' + \operatorname{ch} V'' . \tag{1.1}$$

Further, for $Y, Z \in \mathscr{M}_T$ we have $\operatorname{ch}(Y \otimes Z) = \operatorname{ch} Y . \operatorname{ch} Z$. It is now easy to see that $\mathscr{G}(T)$ is isomorphic to the Grothendieck ring of the category $\mathscr{M}_T$.

Suppose that T_1 and T_2 are tori and φ: $T_1 \to T_2$ is a morphism of algebraic groups. Then φ determines a ring homomorphism $\check{\varphi}$: $\mathscr{G}(T_2) \to \mathscr{G}(T_1)$; for $\lambda \in X(T_2)$, $\check{\varphi}(e(\lambda)) = e(\mu)$, where $\mu(t) = \lambda(\varphi(t))$ for all $t \in T_1$. If T_3 is also a torus and ψ: $T_2 \to T_3$ is a morphism of algebraic groups, then it is easy to see that $(\psi \circ \varphi)\check{} = \check{\varphi} \circ \check{\psi}$.

Now suppose that the torus T is a closed subgroup of an algebraic group G. For $V \in \mathscr{M}_G$ we define $\operatorname{ch}_T(V)$ (or simply $\operatorname{ch} V$ if confusion seems unlikely) to be the character of V considered as a T module. We let

$$\mathscr{P}_G(T) = \{\operatorname{ch} V: \ V \in \mathscr{M}_G\},$$

a subset of $\mathscr{G}(T)$ which is closed under addition and multiplication. The elements of $\mathscr{P}_G(T)$ will be called proper characters.

Let φ: $G \to H$ be a morphism of algebraic groups, T_1 a torus in G and T_2 a torus in G containing $\varphi(T_1)$. For $V \in \mathscr{M}_H$, we define $V^\varphi \in \mathscr{M}_G$ to be the k-space V on which G acts according to the rule $gv = \varphi(g)v$ for $v \in V$, $g \in G$. If α: $Y \to Z$ is a morphism of finite dimensional rational H-modules, then the same map α: $Y^\varphi \to Z^\varphi$ is a morphism of G-modules. Thus φ determines an exact functor $\mathscr{M}_H \to \mathscr{M}_G$.

It is easy to check that

$$\check{\varphi}(\operatorname{ch}_{T_2}(V)) = \operatorname{ch}_{T_1}(V^\varphi). \tag{1.2}$$

Here $\check{\varphi}$ (strictly speaking $\check{\theta}$ where θ: $T_1 \to T_2$ is the map obtained by restricting the domain and codomain of φ) is the ring homomorphism $\mathscr{G}(T_2) \to \mathscr{G}(T_1)$ obtained from φ. Thus we obtain a map

$$\varphi_{\mathscr{P}}: \mathscr{P}_H(T_2) \to \mathscr{P}_G(T_1)$$

by restricting the domain and codomain of $\check{\varphi}$.

LEMMA 1. *If $\varphi|_{T_1}$: $T_1 \to T_2$ is an epimorphism, then $\check{\varphi}$ and $\varphi_{\mathscr{P}}$ are monomorphisms. If $\varphi|_{T_1}$: $T_1 \to T_2$ is a monomorphism, then $\check{\varphi}$ is an epimorphism.*

This may be verified without difficulty.

LEMMA 2. *Let G be an algebraic group, $R_u(G)$ the unipotent radical of G, let $H = G/R_u(G)$ be the quotient group, φ: $G \to H$ the natural map, let T_1 be a torus in G and let $T_2 = \varphi(T_1)$. Then $\check{\varphi}$: $\mathscr{G}(T_2) \to \mathscr{G}(T_1)$ and $\varphi_{\mathscr{P}}$: $\mathscr{P}_H(T_2) \to \mathscr{P}_G(T_1)$ are bijections.*

PROOF. Since $T_2 \cap R_u(G) = 1$, $\varphi|_{T_1}$: $T_1 \to T_2$ is an isomorphism. Hence, by Lemma 1, $\check{\varphi}$ is an isomorphism and $\varphi_{\mathscr{P}}$ is a monomorphism. It remains to show that $\varphi_{\mathscr{P}}$ is surjective. For $V \in \mathscr{M}_G$ we must show that $\operatorname{ch} V = \check{\varphi}(\chi)$ for some $\chi \in \mathscr{P}_H(T_2)$. By (1.1) and induction on the dimension of V it suffices to consider the case in which V is simple. Then, by Clifford's Theorem, V is completely reducible as an $R_u(G)$ module. But the only simple rational module for a unipotent algebraic group is the trivial module (see the theorem of § 17.5 of [3]). Thus $R_u(G)$ acts trivially on V, that is, each element of $R_u(G)$ fixes each element of V.

We define $\bar{V} \in \mathscr{M}_H$ to be the k-space V on which H acts by $xv = gv$ ($x \in H$, $v \in V$), where g is any element of G such that $\varphi(g) = x$. Thus $V = \bar{V}^{\varphi}$ and so, by (1.2),

$$\mathrm{ch}_{T_1}(V) = \check{\varphi}(\mathrm{ch}_{T_2}(\bar{V}))$$

and $\mathrm{ch}_{T_2}(V)$ is in the image of $\varphi_{\mathscr{P}}$.

2. The group G^*.

Now suppose that G is a connected, semisimple, simply connected algebraic group over k and that the characteristic of k is $p \neq 0$. Let T be a maximal torus and let M, Q, and R be as in the statement of the theorem. We define A to be the endomorphism ring $\mathrm{End}_{\boldsymbol{u}_n}(Q)$ of Q. Since the socle of Q is simple, Q is indecomposable and $A/J(A) \approx k$, where $J(A)$ denotes the Jacobson radical of A. An element a of A has a unique expression

$$a = z + a_0 \tag{2.1}$$

with $z \in k$ and $a_0 \in J(A)$. Now $J(A)$ is a subspace amd hence an irreducible closed subset of the affine space $\mathrm{End}_k(Q)$. Thus the subset $U = 1 + J(A)$, obtained by translation by 1, is an irreducible closed subset of $\mathrm{End}_k(Q)$. Since the elements of $J(A)$ are nilpotent linear transformations each element of U is unipotent and in particular has determinant 1. Thus U is a closed, connected unipotent subgroup of the special linear group $\mathrm{SL}(Q)$.

We denote by $\xi \colon M \to Q$ the projection corresponding to the decomposition $M = Q \oplus R$. For $g \in G$ we define $\alpha_g \in \mathrm{End}_k(Q)$ by

$$\alpha_g(y) = \xi(gy)$$

for $y \in Q$. We first note that α_g is non-singular. Let Q' be the kernel of α_g. If $y \in Q'$ and $\gamma \in \boldsymbol{u}_n$ we have

$$\begin{aligned} \alpha_g(\gamma y) &= \xi(g\gamma y) = \xi(\mathrm{Ad}(g)(\gamma)gy) \\ &= \mathrm{Ad}(g)(\gamma)\xi(gy) = \mathrm{Ad}(g)(\gamma)\alpha_g(y) = 0\,, \end{aligned}$$

where $\mathrm{Ad} \colon G \to \mathrm{End}_k(\boldsymbol{u}_n)$ is the adjoint representation of G on $\boldsymbol{u}_n$ (see § 1.3 of [7]). Thus Q' is a $\boldsymbol{u}_n$-submodule of Q and so, if non-zero, has non-zero intersection with the socle Q_0 of Q. However, Q_0 is a G-submodule of M, so

$gy \in Q_0$ for $y \in Q_0$ abd if $y \neq 0$, then $\alpha_g(y) = gy \neq 0$. Hence $Q' \cap Q_0 = 0$ and we must have $Q' = 0$.

We define

$$G = \langle \alpha_g, U: \ g \in G \rangle$$

the subgroup of GL (Q) generated by U and $\{\alpha_g : g \in G\}$. For $g \in G$, $u \in U$, $\gamma \in \boldsymbol{u}_n$, and $y \in Q$,

$$\begin{aligned}
\alpha_g u \alpha_{g^{-1}}(\gamma y) &= \alpha_g u \xi(g^{-1}\gamma y) = \alpha_g u \xi(\mathrm{Ad}\,(g^{-1})(\gamma) g^{-1} y) \\
&= \alpha_g u(\mathrm{Ad}\,(g^{-1})(\gamma)\xi(g^{-1}y)) \\
&= \alpha_g\big(\mathrm{Ad}\,(g^{-1})(\gamma)(u(\xi(g^{-1}y)))\big) \\
&= \xi\big(g\,\mathrm{Ad}\,(g^{-1})(\gamma)(u(\xi(g^{-1}y)))\big) \\
&= \xi\big(\mathrm{Ad}\,(g)(\mathrm{Ad}\,(g^{-1})(\gamma))g(u(\xi(g^{-1}y)))\big) \\
&= \gamma \alpha_g u \alpha_{g^{-1}}(y)\,.
\end{aligned}$$

Hence $\alpha_g u \alpha_{g^{-1}} \in A$ and so, by (2.1), has the form $z + a_0$ for some $z \in k$, $a_0 \in J(A)$. We let

$$Q_1 = \{y \in Q: \ J(A)y = 0\}\,.$$

By Nakayama's Lemma Q_1 is non-zero, moreover it is easily seen to be a $\boldsymbol{u}_n$-submodule of Q and therefore contains Q_0 (since the socle Q_0 is simple). Thus, for $0 \neq y \in Q_0$ we have

$$\alpha_g u \alpha_{g^{-1}}(y) = \alpha_g u(g^{-1}y) = \alpha_g(g^{-1}y) = y\,.$$

On the other hand we have

$$(z + a_0)y = zy$$

so that $z = 1$ and thus $\alpha_g u \alpha_{g^{-1}} \in U$. Hence in particular $\alpha_g \alpha_{g^{-1}} = u'$ for some $u' \in U$ and, for any $u \in U$,

$$\alpha_g u(\alpha_g)^{-1} = \alpha_g u \alpha_{g^{-1}}(u')^{-1} \in U\,,$$

and so U is a normal subgroup of G^*.

We leave the reader to verify the following.

LEMMA 3. *The map φ: $G \to G^*/U$ taking g to $\alpha_g U$ $(g \in G)$ is a surjective group homomorphism.*

Let ψ: $G \to \mathrm{GL}\,(M)$ be the representation of G afforded by M and σ: T

$\to \mathrm{GL}(Q)$ be the representation of T afforded by the T-module Q. (That is, $\sigma(t)(y) = \psi(t)(y)$ for $t \in T$, $y \in Q$). Now σ is a morphism of algebraic groups so $T_1 = \sigma(T)$ is a torus in $\mathrm{GL}(Q)$, and in particular is closed and connected. We put

$$G^{*\prime} = \langle T_1^x, U : x \in G^* \rangle$$

where $T_1^x = xT_1x^{-1}$. Now $G^{*\prime}$ is generated by closed and connected subgroups of $\mathrm{GL}(Q)$ and is therefore closed and connected (by the Proposition of § 7.5 of [3]). However, since G is semisimple, it is generated by the subgroups of G conjugate to T. It follows that the image of φ lies in $G^{*\prime}/U$. Thus, by the surjectivity of φ, we have $G^{*\prime} = G^*$ and G^* is a closed connected subgroup of $\mathrm{GL}(Q)$.

Consider now the determinant function $\delta: G^* \to k^*$. Since U lies in $\mathrm{SL}(Q)$ this factors through U to give a group homomorphism $\bar{\delta}: G^*/U \to k^*$. Composing $\bar{\delta}$ with φ we thus obtain a group homomorphism $G \to k^*$. But G is semisimple and so equal to its derived subgroup. Hence the homomorphism $G \to k^*$ is trivial and so $\bar{\delta}$ and thus also δ is trivial.

We have now shown:

LEMMA 4. *G^* is a closed, connected subgroup of* $\mathrm{SL}(Q)$.

Thus G^* and G^*/U are naturally affine algebraic groups; we wish to show that $\varphi: G \to G^*/U$ is a morphism of algebraic groups. We already know that φ is a group homomorphism and it remains to check that φ is a morphism of varieties. Now φ is the composite of the map $g \mapsto \alpha_g$ $(g \in G)$ with the quotient map $G^* \to G^*/U$, so it suffices to show that the first map $G \to G^*$ is a morphism of varieties. However, G^* is a closed subgroup of $\mathrm{SL}(Q)$ and hence of the affine space $\mathrm{End}_k(Q)$ so it will certainly suffice to check that the map $\alpha: g \mapsto \alpha_g$ $(g \in G)$ from G to $\mathrm{End}_k(Q)$ is a morphism of varieties. We have $\alpha = \theta \circ \psi$, where

$$\theta: \mathrm{End}_k(M) \to \mathrm{End}_k(Q)$$

is defined by $\theta(s)(y) = \xi(sy)$ for $s \in \mathrm{End}_k(M)$, $y \in Q$. Now θ is a linear map and hence a morphism of varieties, also $\psi: G \to \mathrm{End}_k(M)$ is a morphism of varieties and therefore so is the composite α. We have now obtained the following strengthening of Lemma 3.

PROPOSITION. *The map $\varphi: G \to G^*/U$ is an epimorphism of algebraic groups and $U = R_u(G^*)$.*

The group U is equal to the unipotent radical $R_u(G^*)$ of G^* because it is a closed, normal, unipotent subgroup of G^* and, since φ is surjective, G^*/U is semisimple.

We are now ready to prove the theorem. Let $\pi\colon G^* \to G^*/U$ be the natural map, let $\eta\colon T \to T_1$ be the map obtained by restricting the codomain of σ and let $T_2 = \pi(T_1) = \varphi(T)$. We have, as in section 1, ring homomorphisms

$$\check{\pi}\colon \mathscr{G}(T_2) \to \mathscr{G}(T_1), \qquad \check{\varphi}\colon \mathscr{G}(T_2) \to \mathscr{G}(T), \qquad \check{\eta}\colon \mathscr{G}(T_1) \to \mathscr{G}(T)$$

with $\check{\eta} \circ \check{\pi} = \check{\varphi}$ and also maps

$\varphi_{\mathscr{P}}\colon \mathscr{P}_{G^*/U}(T_2) \to \mathscr{P}_G(T)$ and $\pi_{\mathscr{P}}\colon \mathscr{P}_{G^*/U}(T_2) \to \mathscr{P}_{G^*}(T_1)$.

We have

$$\mathrm{ch}_T(Q) = \check{\eta}(\mathrm{ch}_{T_1}(Q)),$$

where Q is regarded as a G^*- (and hence T_1-) module via inclusion $G^* \to \mathrm{End}_k(Q)$. However, by Lemma 2, $\pi_{\mathscr{P}}$ is a bijection and so $\mathrm{ch}_{T_1}(Q) = \pi_{\mathscr{P}}(\chi)$ for some $\chi \in \mathscr{P}_{G^*/U}(T_2)$. Thus we have

$$\mathrm{ch}_T(Q) = \check{\eta} \circ \check{\pi}(\chi) = \check{\varphi}(\chi) = \varphi_{\mathscr{P}}(\chi)$$

and so

$$\mathrm{ch}_T(Q) \in \varphi_{\mathscr{P}}(\mathscr{P}_{G^*/U}(T_2)) \subseteqq \mathscr{P}_G(T)$$

that is, $\mathrm{ch}_T(Q)$ is a proper character.

In order to state and prove the corollary we need some more notation. The Weyl group $W = N_G(T)/T$ acts naturally on $X(T) \otimes_{\mathbf{Z}} \mathbf{R}$; we choose a positive definite, W-invariant, symmetric bilinear form $(\cdot,\cdot)$ on $X(T) \otimes_{\mathbf{Z}} \mathbf{R}$. Let Φ be the root system of G (with respect to T) and Δ a base for the root system, giving a system of positive roots Φ^+ and a system of negative roots Φ^-. For $0 \neq \lambda \in X(T) \otimes_{\mathbf{Z}} \mathbf{R}$ we define $\lambda^\vee = 2\lambda/(\lambda,\lambda)$; we let

$$X^+ = \{\lambda \in X(T)\colon\ (\lambda,\alpha^\vee) \geqq 0 \quad \text{for all } \alpha \in \Delta\} \quad \text{and}$$

$$X_n = \{\lambda \in X^+ :\ (\lambda,\alpha^\vee) < p^n \quad \text{for all } \alpha \in \Delta\}.$$

We have a partial order $\leqq$ on $X(T)$, we decree that $\lambda < \mu$, when $\mu - \lambda$ is a sum of positive roots.

For $\lambda \in X^+$ there is a unique simple G-module $L(\lambda)$ of highest weight λ and, for $\lambda \in X(T)$, a unique simple $\boldsymbol{u}_n$-T-module $\hat{L}(n,\lambda)$ of highest weight λ. Moreover $\{L(\lambda) : \lambda \in X^+\}$ is a full set of simple rational G-modules and $\{\hat{L}(n,\lambda) : \lambda \in X(T)\}$ a full set of simple $\boldsymbol{u}_n$-T-modules. We define the nth Steinberg module St_n to be $L((p^n-1)\varrho)$, where ϱ is half the sum of the positive roots. It is known that St_n is projective as a $\boldsymbol{u}_n$- and as a $\boldsymbol{u}_n$-T-module. We denote by $\hat{Q}(n,\lambda)$, for $\lambda \in X(T)$, the projective cover of $\hat{L}(n,\lambda)$ in the category of $\boldsymbol{u}_n$-T-modules. We define, for $\lambda \in X(T)$, $Q(n,\lambda)$ to be the restriction of $\hat{Q}(n,\lambda)$ to $\boldsymbol{u}_n$ and $L(n,\lambda)$ to be the restriction of $\hat{L}(n,\lambda)$ to $\boldsymbol{u}_n$. Then $\{L(n,\lambda)\colon \lambda \in X_n\}$ is a full set of simple $\boldsymbol{u}_n$-modules and $Q(n,\lambda)$ is the projective cover of $L(n,\lambda)$. Moreover, for $\lambda \in X_n$, the restriction of $L(\lambda)$ to $\boldsymbol{u}_n$-T is isomorphic to $\hat{L}(n,\lambda)$. Further details may be found in [7].

For $\lambda \in X(T)$ we define $\lambda^0 = (p^n-1)\varrho + w_0\lambda$, where w_0 is the longest element of the Weyl group. For $\lambda \in X_n$ we have, by p. 41 of [4] (see also Lemma 5 of [1]) that $L(n,\lambda)$ appears exactly once in the $\boldsymbol{u}_n$-socle of $M = \mathrm{St}_n \otimes L(\lambda^0)$. We let Q_0 be the $\boldsymbol{u}_n$-submodule of M isomorphic to $L(n,\lambda)$. Since the sum of all $\boldsymbol{u}_n$-submodules of M isomorphic to $L(n,\lambda)$ is a G-submodule of M (see § 2.2 of [8]) Q_0 is a G-submodule of M. Moreover, by the Proposition of § 2.3 of [5], we have $Q_0 \approx L(\lambda)$ as a G-module and so $Q_0 \approx \hat{L}(n,\lambda)$ as a $\boldsymbol{u}_n$-T-module. Now $\mathrm{St}_n \otimes L(\lambda^0)$ is projective as a $\boldsymbol{u}_n$-T-module and so a sum of projective indecomposable $\boldsymbol{u}_n$-T-modules. Hence there is a $\boldsymbol{u}_n$-T-summand of M, say Q, containing Q_0 such that Q is a projective indecomposable $\boldsymbol{u}_n$-T-module. Let R be a $\boldsymbol{u}_n$-T complement to Q in M. It follows from [6] that $\hat{Q}(n,\lambda)$- is the injective hull of $\hat{L}(n,\lambda)$ in the category of $\boldsymbol{u}_n$-T-modules and so Q is isomorphic to $\hat{Q}(n,\lambda)$. Hence $\mathrm{ch}\, Q = \mathrm{ch}\, \hat{Q}(n,\lambda)$, moreover M, Q and R satisfy the hypotheses of the theorem. Thus we have:

Corollary. *For $\lambda \in X_n$ the character* $\mathrm{ch}\, \hat{Q}(n,\lambda)$ *of $\hat{Q}(n,\lambda)$ is proper.*

Remark. For each $\lambda \in X^+$ there is a proper character $\chi(\lambda)$ given by Weyl's character formula (see § 5.1 of [7]). It may be that a still stronger property holds. It may be that $\mathrm{ch}\, \hat{Q}(n,\lambda)$, for $\lambda \in X_n$ (or even $\mathrm{ch}\, Q(n,\lambda)\chi(\mu)^F$ for any $\mu \in X^+$, where F is the Frobenius morphism on $\mathbf{Z}X(T)$, as in § 5.1 of [7]) is a sum of the characters $\chi(\tau)$, $\tau \in X^+$. The statement concerning the $\mathrm{ch}\, \hat{Q}(n,\lambda)\chi(\mu)^F$ is true, when p is large by virtue of 5.6 Satz of [8] whereas that concerning $\mathrm{ch}\, \hat{Q}(n,\lambda)$ has been checked, by M. Koppinen, [9], for small primes in several low rank cases.

REFERENCES

1. J. W. Ballard, *Injective modules for restricted enveloping algebras*, Math. Z. 163 (1978), 57–63.
2. E. Cline, *Stable Clifford theory*, J. Algebra 22 (1972), 350–364.
3. J. E. Humphreys, *Linear algebraic groups* (Graduate Texts in Mathematics 21), Springer-Verlag, Berlin - Heidelberg - New York, 1975.
4. J. E. Humphreys, *Ordinary and modular representations of Chevalley groups* (Lecture Notes in Mathematics 528), Springer-Verlag, Berlin - Heidelberg - New York, 1976.
5. J. E. Humphreys, *On the hyperalgebra of a semisimple algebraic group* in *Contributions to Algebra* eds. H. Bass, P. J. Cassidy, J. Kovacic, pp. 203–210, A collection of papers dedicated to Ellis Kolchin, Academic Press, New York - San Francisco - London, 1977.

6. J. E. Humphreys, *Symmetry for finite dimensional Hopf algebras*, Proc. Amer. Math. Soc. 68 (1978), 143–146.
7. J. C. Jantzen, *Über Darstellungen höherer Frobenius-Kerne halbeinfacher algebraischer Gruppen*, Math. Z. 164 (1979), 271–292.
8. J. C. Jantzen, *Darstellungen halbeinfacher Gruppen und ihrer Frobenius-Kerne*, J. Reine Angew. Math. 317 (1980), 157–199.
9. M. Koppinen, *Karakteerikaava Chevalley-ryhmään liittyville hajoamattomille projektiivisille $\mathbf{u}_n$-T moduleille*, Licentiate Thesis, University of Turku, Finland, 1980

KING'S COLLEGE
CAMBRIDGE CB2 1ST
ENGLAND