A REMARK ON RINGS WITH PRIMARY IDEALS AS MAXIMAL IDEALS

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M. Satyanarayana in [1] defines a $P$-ring to be a commutative ring with identity in which every primary ideal is a maximal ideal. Below is a characterisation of such rings.

**Theorem.** A commutative ring $R$ with identity is a $P$-ring if and only if $R$ is regular (in the von Neumann sense).

**Proof.** Let $R$ be a $P$-ring and $M$ any maximal ideal of $R$. The quotient ring $R_M$ has a unique prime ideal $M^e$ other than $R_M$ ([2, Theorem 19, Chapter IV]). Let $I + R_M$ be any ideal of $R_M$. This is a primary ideal; so $I^e$ (see [2] for notation) is a primary and hence a maximal ideal of $R$. Consequently $I^e = M$ whence $I = M^e$. In particular $(0) = M^e$; so that $R_M$ is a field for every maximal ideal $M$ of $R$. Clearly every invertible element of $R$ is regular. So let $r$ be any non-invertible element of $R$ and $T = \{t \in R : rt = 0\}$. This is an ideal of $R$ which is not contained in any maximal ideal $M$, which contains $r$, as $R_M$ is a field. So $T + (r) = R$ which implies $1 = t + rx$ where $t \in T$ and $x \in R$. This gives $r = r^2 x$ and thus $R$ is a regular ring. The other part is immediate.

**References**


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